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EFFECTS OF INCOMPLETE ADAPTION AND DISTURBANCE IN ADAPTIVE CONTROL

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Effects of Incomplete Adaption and Disturbance in Adaptive Control

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I. Introduction

The design of adaptive control systems based upon the application of Liapunov theory has mainly been concerned with the idealized situtation in which the system is free of disturbance [1-3]. The effect of parameter deviations for the case in which the number of adaptive gains is less than the full set required for complete adaption has also been ignored. Based on these simplifications it can be shown that the tracking error of the adaptive system is asymptotically stable.

In recent years efforts have been made to modify the Liapunov design in the interests of practicality and generalization. The main results have been to reduce the number of derivatives of the output variable which need be measured, and to generalize the synthesis procedure to permit more rapid convergence of the tracking error [4]. At least one effort has been made to apply the design to a practical problem [5]. However before the design can be considered to have real engineering significance, the effects of incomplete adaption disturbance and saturation must duly be considered.

The purpose of this paper is to consider the first two of these items with respect to the particular adaptive configuration shown in Figure 1. In order to obtain results which can be readily interpreted, the plant was assumed to be of less general form than is required by the existing theory. However it is evident from the results obtained that stability problems may manifest themselves in the presence of disturbance and adaption errors, and that these problems should not be ignored.

First a sufficient condition for boundedness of the tracking error in the presence of disturbance and adaption errors will be derived. From this result it is shown that boundedness of the tracking error may not in itself quarantee boundedness of the adaptive gain parameters. Hence an independent analysis is required to ascertain the effect of disturbance upon stability of the adaptive gains. It is shown that the disturbance and the input signal can be so related that the adaptive parameters will in fact be unbounded.

A simulation study is carried out with respect to a third-order plant. It is shown in a practical situation that incomplete adaption can lead to a reasonable result, as predicted by the derived error bound. It is further demonstrated that instability due to the action of a disturbance can be brought under control if the input to the system is properly chosen, and that the frequency of the input signal has a significant effect upon the tracking errors.

II. Description of the Adaptive Control System

The time-invariant linear plant to be considered is defined according to the state equation

$$\dot{\underline{x}} = A_p \underline{x} + \underline{b}_p u + \underline{c}_p r + \underline{d} \tag{1}$$

wherein the state vector $\underline{x} = [x_i]$ is of dimension n. $A_p = [a_{ij}]$, $\underline{b}_p = [b_i]$, $\underline{c}_p = [c_n]$ contain constant unknown parameters, and $\underline{d} = [d_i]$ is an unknown bounded disturbance. In assuming that the plant has but one (scalar) input, the possibility of adjusting the plant coefficients $[a_{ij}, b_i]$ directly is ruled out. Hence, as will be seen, adaption is to be obtained by the use of adjustable gains whose outputs act through the control input u so as to cause the plant output to track that of a model. As a consequence of this assumption it can be shown [6] that the state variables in (1) must be chosen as phase

variables, i.e., $\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$, ..., $\dot{x}_{n-1} = x_n$, in which case the following forms for A_p, $\underline{b}_p$ and $\underline{d}$ are obtained:

$$A_p = \left[\begin{array}{c|c} 0 & I \\ \hline a_{n1} & \cdots \quad a_{nn} \end{array}\right], \quad \underline{b}_p = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_n \end{bmatrix}, \quad \underline{d} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ d_n \end{bmatrix}, \quad \underline{c}_p = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ c_n \end{bmatrix}.$$

In the ensuing analysis only the sign of b_n must be known.

The stable time-invariant model, which is assumed to have the same structure as the plant, is defined by,

$$\dot{\underline{y}} = A_m\underline{y} + \underline{b}_m r \tag{2}$$

where $\underline{y} = [y_i]$. $A_m = [\alpha_{ij}]$, $\underline{b}_m = [\beta_i]$ have a form simular to A_p, $\underline{b}_p$ above, and r is a scalar input signal. If the error vector is now defined by

$$\underline{e} = \underline{y} - \underline{x},$$

then the error differential equation can be cast in the form

$$\dot{\underline{e}} = A_m\underline{e} + \underline{f} \tag{3}$$

where

$$\underline{f} = (A_m - A_p)\,\underline{x} + (\underline{b}_m - \underline{c}_p)\,r - \underline{b}_p u - \underline{d} = \Delta\underline{x} + \underline{\delta} r - \underline{b}_p u - \underline{d}. \tag{4}$$

Because of the phase-variable assumption, it can be seen that Δ, δ have the form

$$\Delta = \begin{bmatrix} & 0 & \\ \delta_{n1} & \cdots & \delta_{nn} \end{bmatrix}, \quad \delta = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \delta_n \end{bmatrix}.$$

Therefore $\underline{f}$ has only one non-zero element, namely

$$\underline{f} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ f_n \end{bmatrix}.$$

With reference to Figure 1 the objective is now to design an adaptive controller so as to realize an asymptotic bound on the norm of the tracking error, $||\underline{e}||$, where $\underline{e}$ is goverened by (3). Ideally it is desired that $||\underline{e}||$ should go to zero. In general it is seen that this may require n+1 adaptive gains, however, as will be seen in a case example, such an extravagance may not always be justified because deviation in certain of the plant parameters can have relatively small effect upon the system response, and each adaptive gain increases the level of measurement noise entering the system. The first objective is then to determine a bound on the tracking error when less than n+1 adaptive gains are incorporated in the system, and then to show that a disturbance $\underline{d}$ entering the system can cause instability.

III. Description of the Control Law

According to Parks [1], the design objective will be realized by synthesizing a Liapunov function. Thus starting with the scalar function

$$V = \underline{e}^T P \underline{e} + \underline{\phi}^T \underline{\phi} \tag{5}$$

in which $P = [p_{ij}]$ is to be positive definite symmetric, and $\underline{\phi} = [\phi_i]$ is a parameter dependent vector to be defined, we form the time derivative

$$\dot{V} = \underline{\dot{e}}^T P\underline{e} + \underline{e}^T P\underline{\dot{e}} + 2\underline{\phi}^T \underline{\dot{\phi}} \tag{6}$$

which together with (3) can be written as

$$\dot{V} = \underline{e}^T (A_m^T P + P A_m) \underline{e} + 2(\underline{e}^T P \underline{f} + \underline{\phi}^T \underline{\dot{\phi}}). \tag{7}$$

Using the well-known Liapunov theorem [7], we obtain for any positive-definite symmetric Q, and any stability matrix A_m, a positive-definite symmetric P as a unique solution to the equation

$$-Q = A_m^T P + P A_m \; . \tag{8}$$

The solution proposed by Parks [1] was to select $\underline{\phi}(t)$ so that $\underline{e}^T P \underline{f} + \underline{\phi}^T\underline{\phi} = \underline{0}$.

Thereupon it follows from Liapunov's direct method that V is a Liapunov function, and that the equilibrium at $\underline{e} = \underline{0}$ is asymptotically stable. This result does not, however, allow for certain imperfections and disturbance. Hence in order to define the problem which concerns us here, we write $\underline{f} = \underline{f}_1 + \underline{f}_2$, and require that $\underline{\phi}$ be chosen so that

$$\underline{e}^T P \underline{f}_1 + \phi^T \dot{\phi} \equiv 0. \tag{9}$$

Then (7) together with (8) and (9) becomes

$$\dot{V} = -\underline{e}^T Q\underline{e} + 2\underline{e}^T P \underline{f}_2. \tag{10}$$

Here $\underline{f}_2$ is that part of $\underline{f}$ which by choice or of necessity is not nullified by $\underline{\phi}$. It should be noted that $\dot{V}$ is now indefinite because the sign of $\underline{f}_2$ is not known. To find a bound on $||\underline{e}||$, a spherical region R_e in $\underline{e}$ space must be determined such that $\dot{V} < 0$ for $||\underline{e}|| > R_e$. The derivation for R_e will be deferred to the following section.

At this point it is advantageous to obtain an explicit form for $\underline{f}_2$ and $\underline{\phi}$. For this purpose (10) will be rewritten taking into account that $\underline{f}$ contains only one non-zero element, f_n.

Thus with $f_n = f_{n1} + f_{n2}$, (10) becomes

$$\dot{V} = -\underline{e}^T Q\underline{e} + 2\gamma f_{n2}, \tag{11}$$

in which $\gamma = \sum_{i=1}^{n} P_{in} e_i$ and, according to (9), $\underline{\phi}$ has been chosen so that

$$\gamma f_{n1} + \underline{\phi}^T \dot{\underline{\phi}} \equiv 0. \tag{12}$$

From (4) is is seen that

$$f_n = \sum_{i=1}^{n} \delta_{ni} x_i + \delta_n r - b_n u - d_n. \tag{13}$$

If the control input is now written as

$$u = \sum_{i=1}^{n} k_i x_i + k_{n+1} r \tag{14}$$

then (13) together with (14) can be written as

$$f_n = \sum_{i=1}^{n} (\delta_{ni} - b_n k_i)\, x_i + (\delta_n - b_n k_{n+1})\, r - d_n. \tag{15}$$

We shall now assume that certain elements of the set of gains $[k_i, k_{n+1}]$ are adaptive, and that the remainder are identically zero. In order to satisfy (12), all the terms of f_n in (15) which contain adaptive gains are used to comprise f_{n1}. It follows that (12) is valid if for each $k_i \neq 0$ there is an element ϕ_i of $\underline{\phi}$ such that

$$\gamma(\delta_{ni} - b_n k_i) + \phi_i \dot{\phi}_i = 0 \tag{16}$$

This result will be obtained if

$$\begin{aligned} \phi_i &= \lambda_i (\delta_{ni} - b_n k_i) \\ \dot{\phi}_i &= -\lambda_i b_n \dot{k}_i \end{aligned} \tag{17}$$

where λ_i is an arbitrary non-zero constant, and

$$\dot{k}_i = x_i \gamma / \lambda_i^2 b_n. \tag{18}$$

For the case in which k_{n+1} is adaptive it follows similarly that

$$\dot{k}_{n+1} = r\gamma / \lambda_{n+1}^2 b_n. \tag{19}$$

(14), (18), (19) represent the adaptive control law as derived in [1]. The point of departure from previous work is found in (10) wherein an additional term due to $\underline{f}_2$ appears in the expression for $\dot{V}$.

IV. Derivation of an Error Bound

As a consequence of imperfect adaption and disturbance, boundedness rather than asymptotic stability needs to be investigated. Towards this end, a spherical region R_e is to be found such that $\dot{V} < 0$ for $||\underline{e}|| > R_e$. We are now able to write an explicit form for (11). Recognizing that f_{n2} contains those

parameters which were not identified with adaptive gains, (11) becomes

$$\dot{V} = -\underline{e}^T Q \underline{e} + 2\gamma \left[\sum^m \delta_{ni} x_i + \delta_n r - d_n \right] . \tag{20}$$

Here $\sum^m$ signifies the sum of m terms, but not necessarily in a sequence of successive integers. Thus if m=0, there is an adaptive k_i for every state x_i. If in addition there is an adaptive gain k_{n+1}, then the term $\delta_n r$ does not appear in f_{n2}. It is noted that d_n must always appear in f_{n2}.

It is clear that $\dot{V} < 0$ if $\underline{e}^T Q \underline{e} > 2\gamma f_{n2}$. Thus, choosing Q = I and using $x_i = e_i - y_i$, a spherical region R_e is sought such that, for $||\underline{e}|| > R_e$:

$$\underline{e}^T \underline{e} > 2 \sum_{i=1}^{n} p_{in} e_i \left[\sum^m \delta_{ni} (e_i - y_i) + \delta_n r - d_n \right] . \tag{21}$$

(21) will be satisfied if it is required that

$$\underline{e}^T \underline{e} > 2 \sum_{i=1}^{n} p_{in} |e_i| \left[\sum^m |\delta_{ni}| |e_i| + \sum^m |\delta_{ni}| |y_i| + |\delta_n| |r| + |d_n| \right] \tag{22}$$

Denoting $\max_i |g_i| = |\overline{g_i}|$, and $\max_{i,t} |h_i(t)| = |\tilde{h}_i|$ a stronger condition than (22) is given by

$$\underline{e}^T \underline{e} > 2\bar{p}_{in} \sum_{i=1}^{m} |e_i| \left[|\overline{\delta_{ni}}| \left(\sum^m |e_i| + \sum^m |y_i| \right) + |\bar{\delta}_n| |\tilde{r}| + |\tilde{d}_n| \right] . \tag{23}$$

Using the inequalities [8]

$$\sum_{i=1}^{m} |e_i| \leq (n \sum_{i=1}^{n} e_i^2)^{1/2}$$

$$\sum^m |e_i| \leq (m \sum^m e_i^2)^{1/2} \leq (m \sum_{i=1}^{n} e_i^2)^{1/2}$$

a stronger inequality than (23) can be written as

$$||\underline{e}||^2 > 2\sqrt{n}\, \bar{p}_{in} ||\underline{e}|| \left[|\bar{\delta}_{ni}| (\sqrt{m} ||\underline{e}|| + m |\tilde{y}_i|) + |\bar{\delta}_n| |\tilde{r}| + |\tilde{d}_n| \right] . \tag{24}$$

If

$$(1 - 2\sqrt{nm}\, \bar{p}_{in} |\bar{\delta}_{ni}|) > 0, \tag{25}$$

it follows that $\dot{V} < 0$ for $||\underline{e}|| > R_e$ where

$$R_e = \frac{2\sqrt{n}\ \bar{p}_{in}\ (m\ |\bar{\delta}_{ni}|\ |\tilde{y}_i| + |\bar{\delta}_n|\ |\tilde{r}| + |\tilde{d}_n|)}{1 - 2\ \sqrt{nm}\ \bar{p}_{in}\ |\bar{\delta}_{ni}|} . \tag{26}$$

The condition (25) is a test for the existence of a region $R_e < \infty$, and is a sufficiency condition for stability (boundedness) of the tracking error if, as has been assumed, $\underline{e}^T\ P\ \underline{e}$ in (5) is positive definite. Using the notion that an ultimate bound for $\underline{e}$ must be determined by a contour $\underline{e}^T\ P\ \underline{e}$ = constant circumscribing the sphere of radius R_e, where P is determined by (8) with Q = I, it can be shown that $\underline{e}$ must ultimately reside within a sphere of radius

$$R_e' = \left(\frac{\lambda_{max}}{\lambda_{min}} \right)^{1/2} R_e \tag{27}$$

where λ_{max}, λ_{min} are the max, min eigenvalues, respectively, of the P matrix. Since P is positive definite, $\lambda_{max}/\lambda_{min}$ is a finite positive real number, and R_e' is finite if R_e is finite.

If there is complete adaption (m=0, and $\delta_n r$ does not appear in f_{n2}) and if there is no disturbance ($d_n = 0$), then it is seen that $R_e = 0$, and the system is asymptotically stable in $\underline{e}$ space.

Example of Incomplete Adaption

A case example is introduced here to illustrate how the bound R_e' can be applied to a meaningful problem. Suppose that the model is defined by the transfer function

$$\frac{Y_1}{R}(s) = \frac{1}{(s+1)^2\ (\frac{s}{10} + 1)} = \frac{10}{s^3 + 12s^2 + 21s + 10} \tag{28}$$

and the plant is defined according to

$$\frac{X_1}{U}(s) = \frac{1}{\left(\frac{s}{\omega_1}+1\right)^2\left(\frac{s}{\omega_2}+1\right)}$$

$$= \frac{\omega_1^2\omega_2}{s^3 + (2\omega_1+\omega_2)s^2 + (\omega_1^2 + 2\omega_1\omega_2)s + \omega_1^2\omega_2} . \tag{29}$$

The parameters ω_1, ω_2 are assumed to lie within the ranges of $0.5 < \omega_1 < 1.5$, $9 < \omega_2 < 11$. It is clear that the coefficient $(2\omega_1 + \omega_2)$ has a small percentage variation, although ω_1 can vary by ± 50%. Hence it is reasonable to require that k_1 and k_2 should be adaptive gains, but to assume that $k_3 = 0$ will not lead to excessive tracking errors. In this example the plant parameters were chosen to have the extreme values $\omega_1 = 1.5$, $\omega_2 = 11$.

Assuming phase variable form, it follows in (2) that

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -21 & -12 \end{bmatrix} .$$

With Q = I in (8), the solution for P yields coefficients $p_{13} = 0.05$, $p_{23} = 0.077$, $p_{33} = 0.048$, to be used in generating $\gamma = \sum_{i=1}^{3} p_{i3}e_i$ in (18). For use in calculating the bound R_e' in (27), the eigenvalues of P are determined to be $\lambda = 0.045, 0.926, 2.507$. The adaptive gains were chosen to be

$$k_1 = 10\gamma x_1,\ k_2 = 10\gamma x_2,\ k_3 = 0$$

An input signal r(t) was chosen to be a square wave of unit amplitude and 1/2 sec. period. The disturbance in this case was zero ($\underline{d} = \underline{0}$).

The simulation results shown in Figure 2 portray the resulting errors in $e_1(t)$, $e_2(t)$, $e_3(t)$. The untimate bound on $||\underline{e}||$ for this case is seen

to be $R_e'(\text{actual}) = 1.8$.

The computed bound is found from (26), (27). Thus in this example (27) reduces to

$$R_e = \frac{2\sqrt{n}\,\bar{p}_{in}\,(m\,|\bar{\delta}_{ni}|\,|\tilde{y}_i|)}{1 - 2\sqrt{nm}\,\bar{p}_{in}\,|\bar{\delta}_{ni}|}$$

with $n = 3$, $m = 1$, $\bar{p}_{in} = p_{23} = 0.077$. Since $\delta_{ni} = \delta_{33}$ is the only parameter-deviation term included in f_{n2}, the value for $|\bar{\delta}_{ni}|$ in this case becomes

$$|\bar{\delta}_{ni}| = |\delta_{33}| = |12 - 2\omega_1 - \omega_2| = 2.$$

Also $|\tilde{y}_i| = \max_t\ y_3(t) = 2$. It follows that $R_3 = 2.29$ and $R_3' = 17$. As is to be expected the computed bound offers a conservative estimate.

V. Effect of Disturbance upon Stability

Although the results of the previous section guarantee a bound on the tracking error, the indefiniteness of $\dot{V}$ in (10) due to the presence of $\underline{f}_2$ means that stability in terms of $\underline{\phi}$ is no longer assured, even though $\underline{e}$ is bounded. To illustrate this point, consider the case in which $\underline{e}$ and $\underline{\phi}$ are scalars. In Figure 3a is depicted the solution which results if $\underline{f}_2 = \underline{0}$. Here $\dot{V}$ is semidefinite, and ϕ is bounded. In Figure 3b is depicted the case in which $\underline{f}_2$ is nonzero. Although e is bounded by R_1, it is possible that $|\phi| \to \infty$ as shown. From (17) it is clear that $|k_i| \to \infty$ if $|\phi_i| \to \infty$, a condition which is unacceptable.

We shall consider the case of complete adaption but in which a disturbance is present. The equation (1) for the plant can then be written as

$$\dot{x}_n = \sum_{i=1}^{n} K_i x_i + c_n r + d_n \tag{30}$$

where $K_i = a_{ni} + k_i$, and from (18)

$$\dot{k}_i = c \sum_{j=1}^{n} p_{jn} e_j x_i \tag{31}$$

where c is an arbitrary positive constant.

Since the purpose here is simply to demonstrate that instability can occur, we shall arbitrarily choose r(t) and d(t) as step functions. With these inputs we see that $y \to$ const as $t \to \infty$, and that stability of (30), (31) will result if $\dot{\underline{e}} \to 0$ and $\dot{\underline{K}} \to 0$ (i.e. $\dot{\underline{k}} \to 0$). But $\dot{\underline{K}} \to 0$ requires either $\underline{e} \to 0$ or $\underline{x} \to 0$. The solution $\underline{x} \to 0$ is unacceptable since this requires $||\underline{K}|| \to \infty$. Therefore we examine the conditions which must prevail so that $\underline{e} \to 0$. The equilibrium condition for (30) states that

$$\dot{x}_n = 0 = K_1 x_1 + c_n r + d_n. \tag{32}$$

However a necessary condition for stability at the equilibrium is that all K_i are negative. Therefore it is required from (32) that

$$\lim_{t \to \infty} K_1 = -\frac{(c_n r + d_n)}{x_1} < 0 \tag{33}$$

Let us consider the case in which the model output converges to r, i.e. $y \to r$ as $t \to \infty$. Then with $c_n = 1$, (33) yields

$$\frac{d_n}{r} > -1. \tag{34}$$

If the sign of d_n is not known, (34) can be satisfied if $|r| > |d_n|$. For $|r| < |d_n|$ instability can result, in the sense that $|K_1| \to \infty$ if (34) is not satisfied. This result is significant in that it illustrates the danger of oversimplification when analyzing systems of a complex nature.

Since the results stated above were based on the assumption of constant values for r and d_n, it is worthwhile investigating the behavior of the system with disturbance when r(t) is a time varying function. Then the condition $\dot{\underline{k}} = 0$ will no longer represent an asymptotic solution to (31), and it is possible that stability will no longer depend upon the amplitudes of d_n and r.

Example the Adaption with Disturbance

The system used in the previous example, as defined by (28), (29) will be used in this example to show that the stability problem mentioned above can be avoided if the input signal is time varying. In this case there was complete adaption, i.e. all k_i's were adaptive, d_n was a step function, and the signal r(t) was a square wave. The results in Figure 4 show the variations in $k_1(t)$ for various amplitudes of disturbance, and various square-wave frequencies. The observation is made that $k_1(t)$ is always bounded and that the deviations in $k_1(t)$ become progressively smaller in proportion to the frequency of the square wave input.

The stability problem discussed in this section is considered to be important mainly because it has been ignored. Results [9] have been reported recently which circumvent this problem by constructing $\dot{V}$ so that it is negative definite in $\underline{e}$ and $\underline{\phi}$. For this case it is clear that both $\underline{\phi}$ and $\underline{e}$ will be bounded in presence of disturbance, and for this reason [9] is an important contribution.

VI. Conclusions

The main purpose of this investigation has been to bring to attention the fact that the synthesis of adaptive-control systems has often been discussed in the framework of idealizations which may represent over simplifications. A condition for boundedness of the tracking error has been derived for the case in which incomplete adaption and disturbance are present. However when using Parks'design it is shown that instability of the adaptive gains can result due to the presence of disturbance. The theory has been applied to a non-trivial example in order to illustrate the concepts involved.

Acknowledgement

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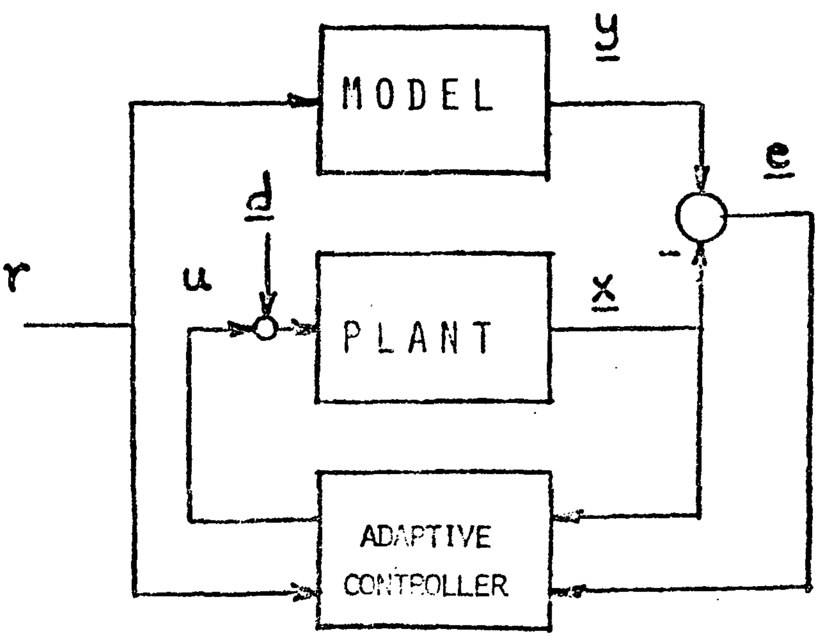


Figure 1

Adaptive System Configuration

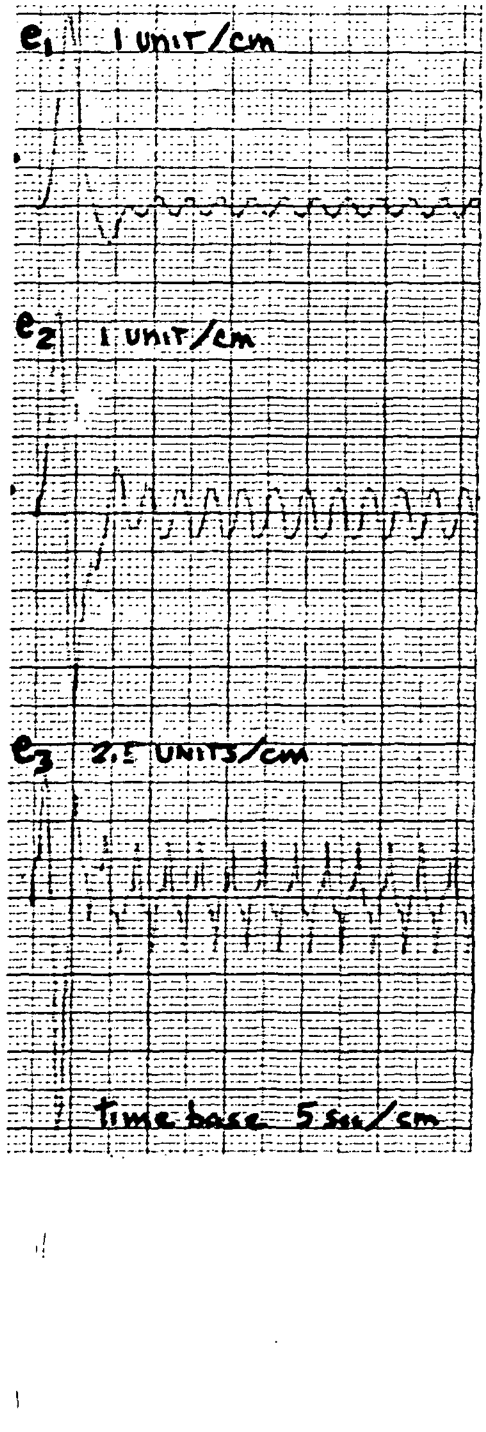
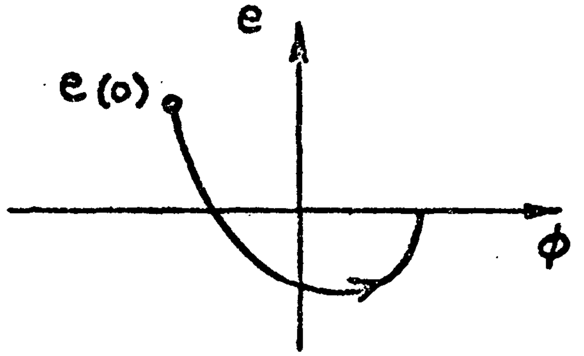
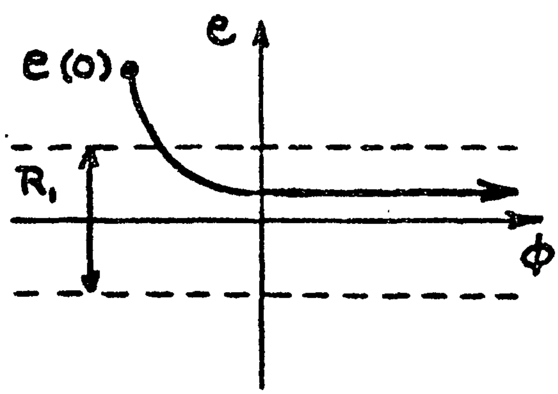


Figure 2

Tracking Errors with Incomplete Adaption



(a) Example of Stability



(b) Example of Instability

Figure 3

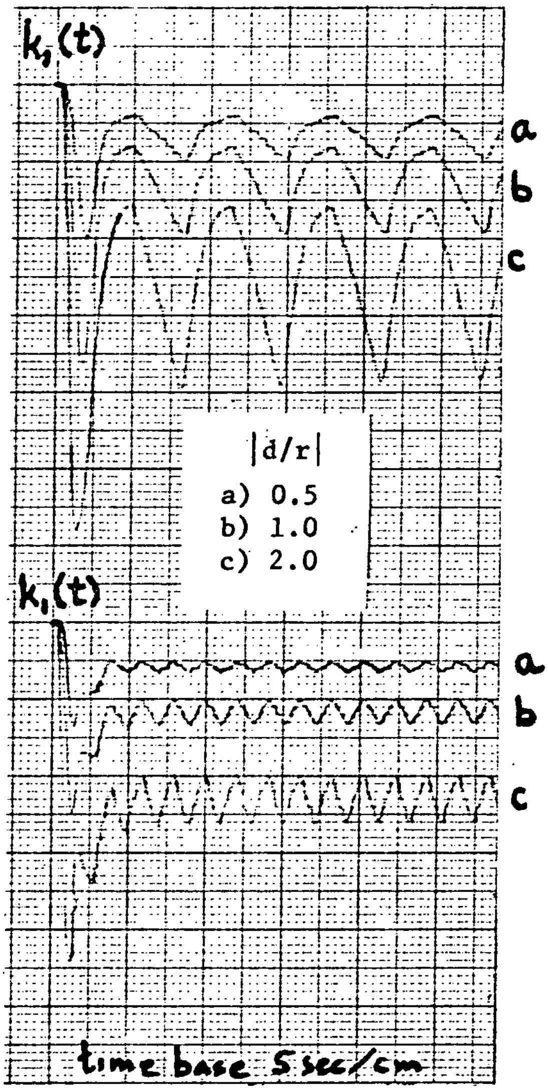


Figure 4

Adaptive-Gain Variation with Disturbance